We prove that every  $C^1(\bar{\omega})$ -regular subsolution of the Monge-Ampere system posed on a 2-dimensional domain  $\omega$  and considered in codimension k = 2, can be uniformly approximated by its exact solutions with regularity  $C^{1,\alpha}(\bar{\omega})$  for any  $\alpha < 1$ . This asserts flexibility of Poznyak's theorem for isometric immersions of 2d Riemannian manifolds into  $\mathbb{R}^4$ , in the parallel setting of the Monge-Ampere system.